Evaluation of Four Methods to Estimate Parameters of an Annual Tree Survival and Diameter Growth Model

Quang Cao and Mike Strub

Abstract: An approach to simultaneously estimate parameters of an annual tree growth model was developed, in which the sum of log-likelihood functions for tree survival and diameter growth was maximized. Four methods for acquiring interim values of stand density were evaluated: (1) Updating Attributes, in which individual tree values were summarized at the end of each year within the growth period to predict interim stand-level attributes, (2) Predicting Attributes, in which stand attributes were predicted annually using a stand-level model, (3) Linear Interpolation, in which stand attributes were predicted by linear interpolation, and finally (4) Initial Values, in which stand attributes at the beginning of the growth period were used as predictors throughout the growing period, and the rate of change for tree survival and diameter was assumed to be constant for this period. Data from the Southwide Seed Source Study of loblolly pine (Pinus taeda L.) showed that, overall, the Updating Attributes and Predicting Attributes produced better evaluation statistics in predicting tree survival and diameter growth than did the other two methods. A simulation study confirmed that these two methods produced the least biased estimates of parameters. Compared with the Updating Attributes method, the Predicting Attributes method produced similar evaluation statistics in predicting tree survival and diameter growth and similar bias in estimating model parameters. The Predicting Attributes method therefore offers a reasonable alternative to the Updating Attributes method, because of the ease of programming in available software languages. For. Sci. 54(6):617–624.

Keywords: annual prediction, individual tree model, simultaneous estimation, maximum likelihood estimation, Pinus taeda


The problem of estimating parameters of the annual tree growth model becomes more complicated because trees are often not measured every year but at some interval, which might vary from plot to plot even in the same study. The standard method until recently to handle this problem has been to assume a constant rate of tree survival and growth during the entire growing period and to use stand attributes at the beginning of the growth period for predicting average annual increment. This assumption is too simplistic because as time passes, both stand variables (stand height, density, and other) and tree variables (diameter and height) change and, as a result, tree survival and diameter growth should vary from year to year.

Improvement to the constant rate method started with interpolation methods developed by McDill and Amateis (1993) for modeling one variable (e.g., tree height) and later generalized by Cao et al. (2002) for many variables (tree diameter, height, and crown ratio). The interpolation methods above still assumed a constant survival rate for the growth period. Cao (2000) introduced an iterative method to account for variable rates of both tree survival and diameter growth. In this method, survival probability and diameter of each tree in the plot were predicted, and interim values of stand density (number of trees and basal area per ha) were updated for each year. Nord-Larsen (2006) used a similar method for modeling tree growth data with highly irregular measurement intervals.

Updating interim values of stand density annually from predicted tree survival and diameter growth is cumbersome in estimating the regression parameters and also in applying the final model. Cao (2002, 2004) alleviated this problem by using a stand-level model to predict intermediate values of stand density annually throughout the growing period.

The objective of this study was to evaluate four methods for estimating parameters of a tree survival and diameter growth system, in which interim annual stand density was updated either by summing intermediate tree attributes, from a stand-level model, from interpolated values, or from initial conditions. A maximum likelihood approach was developed to simultaneously estimate parameters of both equations in the growth system for all methods.

Data

Data used in this study were from loblolly pine (Pinus taeda L.) plantations in the Southwide Seed Source Study.
which included 15 seed sources planted at 13 locations across 10 southern states (Wells and Wakeley 1966). Each plot consisted of 49 trees planted at a 1.8 × 1.8 m spacing. Plot area was therefore approximately 0.0164 ha. Tree diameters were measured at ages 10, 15 (or 16), 20 (or 22), and 25 (or 27) years.

The fit data set, consisting of 100 plots, was randomly selected from the original data. Four methods of estimating parameters of the tree model were applied on consecutive growth periods from the fit data (total of 300 growth periods), ranging from 4 to 7 years. The validation data sets were formed by randomly selecting 100 plots from the remaining plots. All possible growth periods from the validation data (total of 600 growth periods), ranging from 4 to 17 years, were used for evaluating the two methods. Table 1 shows distribution of these plots by measurement age. Summary statistics of stand and tree variables are presented in Table 2 for the fit and validation data sets.

### Methods

Forest modelers have generally described individual tree survival and growth as functions of stand attributes (stand age, site quality, and stand density), tree attributes (tree diameter and height), and some measure of competition (e.g., total basal area of trees having larger diameters). The following system of equations was adopted after preliminary analyses to predict tree annual survival and diameter growth, respectively:

\[
p_{i,t+1} = \left[1 + \exp\left(\alpha_i + \frac{\alpha_t H_t}{A_i} + \alpha_d R_t + \frac{\alpha_{d,i} d_{i,t}}{D_{q,t}}\right)\right]^{-1}, \tag{1a}
\]

\[
\hat{d}_{i,t+1} = \hat{d}_{i,t} + \exp\left[\beta_i + \frac{\beta_t H_t}{A_i} + \beta_d R_t + \beta_{d,i} \text{bal}_{i,t}\right]
+ \beta_{d,i} \ln(\hat{d}_{i,t}), \tag{1b}
\]

where \(p_{i,t+1}\) is the probability that the \(i\)th tree survives at time \((t + 1)\), given that it was alive at time \(t\); \(A_i\) is stand age in years at time \(t\); \(H_t\) is average height of the dominants and codominants in \(m\) at time \(t\), predicted from \(H_t = \exp[\alpha_i + \ln(H_{t-1}) - \alpha_i(A_{t-1}/A_i)]\); \(R_t\) is stand density at time \(t\) in terms of number of trees per ha; \(\hat{d}_{i,t}\) is stand density at time \(t\) in terms of basal area (\(m^2/ha\)); \(d_{i,t}\) is dbh in cm of the \(i\)th tree at time \(t\); \(D_{q,t}\) is the quadratic mean diameter in cm at time \(t\); \(\text{bal}_{i,t}\) is basal area in \(m^2/ha\) at time \(t\) of the trees having diameters larger than \(\hat{d}_{i,t}\); \(\ln(\cdot)\) is the natural logarithm; and \(\alpha, \beta, \lambda\) are regression coefficients.

Four methods for deriving interim values when estimating parameters of the tree growth and survival models were evaluated: (1) Updating Attributes, in which individual tree values were summarized at the end of each year within the growth period to predict interim stand-level attributes; (2) Predicting Attributes, in which stand attributes were predicted annually using a stand-level model; (3) Linear Interpolation, in which stand attributes were predicted by linear interpolation; and finally (4) Initial Values, in which stand attributes at the beginning of the growth period were used as predictors throughout the growing period, and the rate of change for tree survival and diameter was assumed to be constant for this period.

### Method 1: Updating Attributes

In this method, survival probability and diameter of each tree in the plot were predicted annually. Interim values of stand density were updated for each year from

\[
N_{i,t+1} = (1/s) \sum p_{i,t,i}, \quad \text{and} \tag{2a}
\]

\[
B_{i,t+1} = (K/s) \sum p_{i,t,i} \hat{d}_{i,t}, \tag{2b}
\]

where \(K = \pi/40,000\) is a factor to convert diameter from cm into basal area in \(m^2\), \(s\) is plot size in ha, and the summation sign includes all trees in the plot.

Annual values of \(\text{bal}_{i,t}\) for tree \(i\) were updated from Equation 2b, except that the summation sign in this case included only trees having diameters larger than \(\hat{d}_{i,t}\). \(\text{D}_{q,t}\) was calculated as the square root of the average of the \(\hat{d}_{i,t}\) squared.

### Method 2: Predicting Attributes

Instead of updating interim values of stand density by annually summarizing plot data, the following stand-level model was used to predict intermediate values of stand density throughout the growing period:

\[
N_{i,t+1} = \exp\left[\frac{A_i}{A_{t+1}} \ln(N_i) + \left(1 - \frac{A_i}{A_{t+1}}\right)\left(\gamma_1 + \frac{\gamma_2}{A_i} + \gamma_3 N_i\right)\right], \tag{3a}
\]

\[
\text{D}_{q,t+1} = \exp\left[\frac{A_i}{A_{t+1}} \ln(D_{q,t}) + \left(1 - \frac{A_i}{A_{t+1}}\right)\left(\delta_1 + \frac{\delta_2}{A_i} + \delta_3 R_t\right) + \delta_4 \text{D}_{q,t}\right], \tag{3b}
\]

Basal area per ha of trees larger than the \(i\)th tree in terms of diameter was projected annually as

\[
\text{bal}_{i,t+1} = \text{bal}_{i,t} + \exp\left[\kappa_1 + \frac{\kappa_2}{A_i} + \frac{\kappa_3 H_t}{A_i} + \kappa_4 R_t + \kappa_5 N_i + \kappa_6 \text{bal}_{i,t}\right]. \tag{4}
\]

### Table 1. Distribution of plots in the fit and validation data set, by measurement age

<table>
<thead>
<tr>
<th>Measurement ages (yr)</th>
<th>Fit</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10, 15, 20, 25</td>
<td>61</td>
<td>64</td>
</tr>
<tr>
<td>10, 15, 20, 27</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>10, 15, 22, 27</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>10, 16, 20, 27</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

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The probability of the $i$th tree surviving the $q$-year growing period, where $q$ varied from 4 to 7 years in this study, is the product of annual probabilities:

$$plive_i = \prod_{j=1}^{t+q} P_{i,j}.$$  (7)

Equation 7 can be considered as a modified form of logistic regression. Maximum likelihood estimators can be obtained either indirectly (Cao 2000, 2002, 2004) by using the weighted least-squares approach or directly by maximizing the log-likelihood function (Nord-Larsen 2006). Flewelling and Monsrud (2002) recommended either a weighted nonlinear least-squares method or a maximum likelihood method for unequal period lengths. The log-likelihood function in this case is

$$\ln(L_i) = \sum_{j=1}^{n_i} z_i,$$  (8)

where $n_i$ is the total number of trees in the fit data set and $z_i = \ln(plive_i)$ if tree $i$ was alive at the end of the period and $\ln(1 - plive_i)$ otherwise.

For tree diameter, Equation 1b was used repeatedly to produce $\hat{d}_{i,t+q}$, which predicted diameter of tree $i$ at the end of the growth period ($d_{i,t+q}$):

$$d_{i,t+q} = \hat{d}_{i,t+q} + \varepsilon_i,$$  (9)

where $\varepsilon_i$ = error, assumed to be normally distributed with mean 0 and variance $\sigma^2$. The log-likelihood for Equation 9 is as follows:

$$\ln(L_i) = -n_i \ln(\sigma) - n_i^2 \sigma^{-2} \ln(2\pi)$$

$$- \frac{1}{2\sigma^2} \sum_{j=1}^{n_i} (d_{i,t+q} - \hat{d}_{i,t+q})^2,$$  (10)
where \( n_2 \) is the total number of trees in the fit data set that survived the entire growing period.

We propose to simultaneously estimate the parameters of both Equations 7 and 9 using the maximum likelihood technique. The combined likelihood function is motivated by considering the following bivariate distribution of tree diameter growth and survival:

\[
P(X_i, Y_i) = \begin{cases} 
P(X_i | Y_i = 1) \cdot P(Y_i = 1), & \text{if } Y_i = 1, \\
P(X_i | Y_i = 0) \cdot P(Y_i = 0), & \text{if } Y_i = 0, 
\end{cases}
\] (11)

where \( X_i \) is the diameter growth of tree \( i \) during the period, and \( Y_i = 1 \) if tree \( i \) survives the period and \( 0 \) otherwise.

In the case of mortality, we assume that the tree died at the beginning of the growing period and there was no diameter growth. Therefore \( X_i = 0 \), resulting in \( P(X_i | Y_i = 0) = 1 \). Equation 11 can be rewritten as

\[
\ln[P(X_i, Y_i)] = \begin{cases} 
\ln[P(X_i | Y_i = 1)] + \ln[P(Y_i = 1)], & \text{if } Y_i = 1, \\
\ln[P(Y_i = 0)], & \text{if } Y_i = 0, 
\end{cases}
\] (12)

which leads to

\[
\ln(L) = \ln(L_2) + \ln(L_1). 
\] (13)

An alternate justification is if errors in mortality prediction and diameter growth estimation are independent of each other, then the log-likelihood equation for both events is the sum of the two log-likelihood equations.

The parameters of both Equations 7 and 9 were simultaneously estimated by maximizing \( \ln(L) \), or minimizing

\[
-\ln(L) = n_2 \ln(\sigma) + \frac{n_2}{2} \ln(2\pi) \\
+ \frac{1}{\sigma^2} \sum_{i=1}^{n_2} (d_{i,t+q} - \hat{d}_{i,t+q})^2 \\
- \sum_{i=1}^{n_2} z_i. 
\] (14)

In each iteration, the variance, \( \sigma^2 \), was estimated from

\[
\left( \frac{1}{n_2 - k_2} \right) \sum_{i=1}^{n_2} (d_{i,t+q} - \hat{d}_{i,t+q})^2, 
\]

where \( k_2 = 5 \) is the number of parameters in Equation 1b, and \( \hat{d}_{i,t+q} \) is the predicted value for \( d_{i,t+q} \) from the previous iteration.

SAS procedure NLIN (SAS Institute, Inc., 2004) was used to solve the minimization problem of Equation 14. Let us define \( y_i = 0 \) and

\[
\hat{y}_i = \left( -\frac{1}{z_i} + I_{i,t} \left( \ln(\sigma) + \frac{(d_{i,t+q} - \hat{d}_{i,t+q})^2}{2\sigma^2} \right) \right), 
\]

where \( I_{i,t} = 1 \) if tree \( i \) survived at age \( A_{i,t+q} \) and \( 0 \) otherwise. Minimizing the sum of squared errors or \( \sum (y_i - \hat{y}_i)^2 \) was equivalent to minimizing \( -\ln(L) \) and yielded maximum likelihood estimates of the parameters of the growth system. The algorithm used to compute \( \hat{y}_i \) for the \( i \)th tree is shown in the Appendix. The Initial Values approach (method 4) directly predicts survival and growth for a measurement interval and therefore does not require the logic described in this algorithm.

Evaluation

Short growth periods (4–7 years) based on consecutive growing periods in the fit data set were used for estimating the parameters. On the other hand, evaluation from the validation data involved all possible growing periods, including short (4–7 years), medium (10–12 years), and long (15–17 years) projection periods. The intention was to determine how well the model extrapolated beyond the data range. Criteria used in evaluating predictions of tree survival probability were mean difference (between observed and predicted values) and mean absolute difference. For tree diameters, the evaluation statistics were mean difference, mean absolute difference, and fit index (which is computationally identical to \( R^2 \) in linear regression).

Results and Discussion

Parameter estimates and their standard errors for Equations 3 and 4 are presented in Table 3. These equations were used to predict interim values of \( N_{t+1}, \hat{D}_{q,t+1} \), and \( \text{bal}_{t+1} \) to be used in the Predicting Attributes method.

Table 4 shows parameter estimates and their standard errors for the tree survival and diameter growth system from all four methods. All parameters presented in Table 4 were significant at the 5% level. The Akaiane information criterion values from all methods were similar, with methods 3 and 4 being slightly better than methods 1 and 2.

Evaluation statistics for predicting tree survival (Figure 1) included mean difference between observed and predicted survival probabilities and mean absolute difference. An unexpected result was that the bias, or mean difference, values were smaller for long projection periods compared with those for medium projection lengths. On average, all methods overpredicted tree survival, except for the Initial Values method at short projections. Method 3 (Linear Interpolation) produced the smallest mean absolute difference values. Evaluation statistics for methods 1 and 2 were

| Table 3. Parameter estimates for Equations 3 and 4 |
|-----------------|-----------|----------|---|
| Equation | Parameter | Estimate | SE |
| 3a | \( \gamma_1 \) | 3.9467 | 0.2096 |
| 3a | \( \gamma_2 \) | 32.4557 | 3.3179 |
| 3a | \( \gamma_3 \) | 0.0002 | 0.0001 |
| 3b | \( \delta_1 \) | 3.1544 | 0.1366 |
| 3b | \( \delta_2 \) | -8.1406 | 1.0025 |
| 3b | \( \delta_3 \) | 1.5231 | 0.2615 |
| 3b | \( \delta_4 \) | 0.0291 | 0.0045 |
| 4 | \( \kappa_1 \) | 15.5784 | 1.3973 |
| 4 | \( \kappa_2 \) | 78.8367 | 4.4454 |
| 4 | \( \kappa_3 \) | -5.3739 | 0.3801 |
| 4 | \( \kappa_4 \) | -11.6949 | 1.2830 |
| 4 | \( \kappa_5 \) | -2.0285 | 0.1557 |
| 4 | \( \kappa_6 \) | 0.0344 | 0.0015 |

Equation form:

(3a) \( \hat{N}_{t+1} = \exp[A_i \hat{A}_t \ln(\hat{N}_t)] + (1 - A_i \hat{A}_t)\gamma_1 + \gamma_2 \hat{A}_t + \gamma_3 \hat{N}_t \)

(3b) \( \hat{D}_{q,t+1} = \exp[A_i \hat{A}_t \ln(D_q) + (1 - A_i \hat{A}_t)\delta_1 + \delta_2 \hat{A}_t + \delta_3 \hat{RS}_t + \delta_4 \hat{D}_q] \)

(4) \( \text{bal}_{t+1} = \text{bal}_t + \exp[k_1 + k_2 \hat{A}_t + k_3 \hat{R}_t / \hat{A}_t + k_4 \hat{RS}_t + k_5 \hat{N}_t] \)
Table 4. Parameter estimates for the tree survival and diameter growth system, by method

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
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<td>(\beta_5)</td>
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</tr>
</tbody>
</table>

\(\text{AIC, Akaike information criterion.}

\begin{align*}
1a & \quad \hat{\rho}_{i, t+1} = \left[ 1 + \exp(\alpha_1 + \alpha_2 A_i + \alpha_3 R_{i, t} + \alpha_4 \hat{d}_{i, t} + \alpha_5 D_{i, t} \right]^{-1}
2b & \quad \hat{d}_{i, t+1} = \hat{d}_{i, t} + \exp(\beta_1 + \beta_2 A_i + \beta_3 R_{i, t} + \beta_4 \hat{d}_{i, t} + \beta_5 \ln(d_{i, t})).
6a & \quad \hat{\rho}_{i, t+1} = \left[ 1 + \exp(\alpha_1 + \alpha_2 A_i + \alpha_3 R_{i, t} + \alpha_4 \hat{d}_{i, t} + \alpha_5 d_{i, t} \right]^{-1}
6b & \quad \Delta d = \exp(\beta_1 + \beta_2 A_i + \beta_3 R_{i, t} + \beta_4 \hat{d}_{i, t} + \beta_5 \ln(d_{i, t})).
\end{align*}

\(\text{similar, with method 1 (Updating Attributes) being slightly better.}

The last two methods, based on constant rate (Initial Values) and interpolation (Linear Interpolation), estimate parameters by using observed independent variables. Barreto and Howland (2006) showed that when these methods are applied to simultaneous equations (in this case survival and growth), biased parameter estimates result. They label this simultaneity bias. Simultaneity bias occurs when a system of two or more equations contains the dependent variable from one equation as an independent variable in another equation. They show that methods that use estimated (from other equations in the system of simultaneous equations) independent variables result in unbiased parameter estimates. These results are so well known in the econometrics literature that accounts of the history of the discovery of this bias have been reported in Morgan (1990).

The Updating Attributes and Predicting Attributes methods rely on estimated independent variables and therefore should not have simultaneity bias. Results from this study confirmed that bias was indeed less for these two methods.

Figure 2 presents evaluation statistics for predicting tree diameters computed from the validation data set for different projection lengths. These statistics are mean difference between observed and predicted diameters, mean absolute difference, and fit index. Methods 1 and 2 are close, outscoring the other two methods in all cases. The Initial Values method was consistently the worst among all methods. Again as expected, bias was less for the first two methods that rely on estimated independent variables and should not have simultaneity bias.

Table 5 shows results of Duncan’s multiple range test performed on the overall mean of the evaluation statistics from the four methods (with projection periods and trees considered as blocks). On the basis of all evaluation statistics, the four methods were classified into three distinct groups that were significantly different from one another at the 5% level. The overall means from methods 1 and 2 are not significantly different; whereas they were significantly different from those produced by methods 3 and 4.

**Figure 1. Evaluation statistics based on tree survival probability computed from the validation data set for different projection lengths. These statistics are (a) mean difference and (b) mean absolute difference. The four methods evaluated are (1) Updating Attributes, (2) Predicting Attributes, (3) Linear Interpolation, and (4) Initial Values.**

**Separate Estimation of Parameters**

Alternative methods were explored for estimating parameters of the tree survival and diameter growth equations separately. Least-squares techniques were used to estimate...
parameters of the diameter growth Equation 1b, whereas maximum likelihood estimation of parameters of the survival Equation 1a was obtained through weighted nonlinear regression. The Updating Attributes method required an iterative procedure outlined by Cao (2000), in which new parameter estimates for one equation were determined by use of the current set of parameters for the other equation. The iterative procedure converged quickly after four iterations. In the Predicting Attributes method, parameters from the diameter growth equation were estimated first, followed by those from the survival equation. This was similar to the two-step procedure developed by Cao (2002).

These alternative methods of separate estimation yielded parameter estimates that were similar to those obtained from simultaneous estimation and produced evaluation results that were almost identical to those from simultaneous estimation.

Simulation Study

Whether or not a method is appropriate for estimating parameters of a model depends on whether the model is sufficiently specified and whether eccentricities were present (e.g., outliers) in the data or both. Monte Carlo analysis was used in this study to ensure that the model was correctly specified and that the four methods could be reliably and accurately evaluated. The method that provided estimates closest to the true (known) parameters was considered the best method.

Simulation growth data were generated from the fit data set, using Equations 1a and 1b for predicting annual changes in tree survival and diameter. Coefficients from the Updating Attributes method (Table 4) were used for this procedure. A total of 500 simulated data sets were generated. Each data set contained initial measurements from the 100 plots in the fit data as the initial data. At the end of each year in the growing period (varying between 4 and 7 years), a tree survived if a uniform (0, 1) random number was less than the survival probability computed for that tree from Equation 1a. Diameter growth of each surviving tree was then computed every year by adding a random term to the predicted annual growth from Equation 1b. The random error term was from a normal distribution with mean zero and variance equal to the mean squared error from Equation 1b. The random error was truncated on both sides at plus or minus predicted annual growth to ensure that positive diameter growth was attained and also that the error distribution remained symmetrical.

Figure 3 shows that methods 1 (Updating Attributes) and 2 (Predicting Attributes) produced estimators that were the least biased (<7%), with method 1 slightly edging method 2. As expected, large bias up to 30 and 60% resulted from method 3 (Linear Interpolation) and method 4 (Initial Values) method, respectively, owing to simultaneity bias.

Updating versus Predicting Attributes
Programming Consideration

The Predicting Attributes method is relatively straightforward and can be easily applied using available statistical software such as SAS proc NLIN. The Updating Attributes is much more difficult to program using NLIN, requiring the use of an array of all tree diameters in a plot for each observation. A similar programming procedure was shown by Strub and Cieszewski (2002) in dealing with estimating parameters of a site index equation.

Furthermore, the Updating Attributes method consumed significant time in summarizing plot information at the end of each year. This method required more than eight times the CPU time needed for the Predicting Attributes method (42.96 versus 5.01 seconds).

Interim Stand Attributes

Resulting stand attributes (number of trees per ha and quadratic mean diameter) at the end of the growth period were similar for both methods, clustering about the 1:1 line. For basal area per ha of trees larger than a subject tree (bal), the Predicting Attributes method matched the Updating Attributes method for small to medium bal, but tended to underpredict for larger bal values. This may be because Equation 4 did not include tree diameter as an independent variable.
Table 5. Overall mean of evaluations statistics for the four methods

<table>
<thead>
<tr>
<th>Evaluation statistic*</th>
<th>Parameter estimating method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Updating attributes</td>
</tr>
<tr>
<td>Tree survival</td>
<td></td>
</tr>
<tr>
<td>Mean difference</td>
<td>-0.0112&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Mean absolute difference</td>
<td>0.3204&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Tree diameter</td>
<td></td>
</tr>
<tr>
<td>Mean difference</td>
<td>0.1804&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>Mean absolute difference</td>
<td>1.3540&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

* For each evaluation statistic, means with the same letter in the same row are not significantly different at the 5% level from Duncan’s multiple-range test (with projection periods and trees as blocks).

Figure 3. Percent bias in estimating 10 regression parameters using four methods: (1) Updating Attributes; (2) Predicting Attributes; (3) Linear Interpolation; and (4) Initial Values. Average bias was computed from a simulation study involving 500 replications.

Applying the Model

To be consistent, the model should be applied using the same method from which its parameters were obtained. For example, if the Updating Attributes method was used for parameter estimation, then stand attributes in the intermediate years within the growing period should also be updated from the plot summary when it is used to make predictions. However, results showed that using the wrong method when applying the model did not hurt the performance of the model. Similar evaluation statistics were obtained regardless of which method was used in applying the model.

Conclusions

Simultaneous estimation of parameters of the annual tree growth model was obtained by maximizing the sum of log-likelihood functions for tree survival and diameter growth. Overall, the Updating Attributes and Predicting Attributes methods produced better evaluation statistics in predicting tree survival and diameter growth than did the other two methods. Results from an additional simulation study showed that these two methods produced the least biased estimates of parameters. Compared with the Updating Attributes method, the Predicting Attributes method produced similar evaluation statistics in predicting tree survival and diameter growth and similar bias in estimating model parameters. The Predicting Attributes method runs faster and is easier to program in available software languages and therefore offers a reasonable alternative to the Updating Attributes method.

Literature Cited


Agriculture, Forest Service, Rocky Mountain Research Station, Ogden, UT.


Appendix

Algorithm for computing $\hat{y}_i$.

Loop for $i$ from $A_{t+1}$ to $A_{t+q}$

Loop for tree $j$ from 1 to $n_1$

Compute $\hat{d}_{ij}$ and $p_{ij}$

End {loop for tree $j$}

Compute $N_{t+q}$ and $B_{t+q}$:

Method 1: Updating attributes from plot summary

Method 2: Predicting attributes from Equation 3

Method 3: Interpolating intermediate attributes

End {loop for $i$}

Compute $\hat{y}_i$.

where $I_{i,t}$ is 1 if tree $i$ survived at the end of the growth period and 0 otherwise; $\sigma^2$ is the variance of $e_i$ from $d_{i,t+q} = \hat{d}_{i,t+q} + e_i$; $d_{i,t+q}$ and $\hat{d}_{i,t+q}$ are observed and predicted tree diameter at the end of the growth period; $zi = \ln(\text{plive}_i)$ if tree $i$ was alive at the end of the period and $\ln(1 - \text{plive}_i)$ otherwise; and $\text{plive}_i$ is the probability that tree $i$ survived at the end of the period.