Use of Modified Reineke’s Stand Density Index in Predicting Growth and Survival of Chinese Fir Plantations

Xiongqing Zhang, Quang V. Cao, Lele Lu, Hanchen Wang, Aiguo Duan, and Jianguo Zhang

Stand density index (SDI) has played an important role in controlling stand stocking and modeling stand development in forest stands. Reineke’s SDI (SDI_R) is based on a constant slope of –1.605 for the self-thinning line. For Chinese fir plantations, however, it has been reported that the self-thinning slope varied with site and climate, rendering SDI_R questionable. Remeasured data from 48 plots distributed in Fujian, Jiangxi, Guangxi, and Sichuan provinces were used to develop models for prediction of stand survival and basal area, with SDI_R incorporated as a predictor variable. Also included in the evaluation were growth models based on self-thinning slopes estimated from two groups of sites (SDI_S) or from climate variables (SDI_C). Results indicated that models with climate-sensitive SDI (SDI_C) performed best, followed by SDI_S and SDI_R. The control models without SDI received the worst overall rank. Inclusion of climate-sensitive SDI in growth and survival models can therefore facilitate modeling of the relation between stand density and growth/survival under future climate-change conditions.

Keywords: site-specific SDI, climate-sensitive SDI, growth, survival

Stand density has been recognized as an important variable for reflecting stocking levels in forest management (Stout and Larson 1988), because it has a critical influence over recruitment, growth, and mortality at a given location (Curtis 1970, Gleason et al. 2017). Effective and simple indicators for stand density are required for modeling the growth and yield of forest stands. The simplest measure of number per unit area is necessary but not sufficient to adequately describe stand density. Furthermore, tree size should also be a vital component in a stand density measure. Several better-known indicators include relative spacing (Hart 1926), stand basal area, the S-curve (O’Connor 1935), SDI (Reineke 1933), and the –3/2 power law (Yoda 1963). Reineke (1933) defined the SDI according to the relation between the number per unit area and the quadratic mean diameter ($D_q$) of stands. Yoda et al. (1963) related the mean plant biomass (or volume) to the number per unit area. Zeide (1987) suggested using Reineke’s equation rather than Yoda et al.’s because diameter had a better correlation with crown width. In addition, tree diameter, the most practical size variable for management, can be obtained more accurately and easily at little cost compared to biomass (Mdel et al. 2001).

Reineke (1933) concluded that maximum size–density relation had a linear trend with a universally fixed slope in even-aged, fully stocked forest stands, despite tree species and site. However, instead of being a constant, slope has been found to vary by species

Growth models for even-aged forest stands usually involve age, stand density, site index, and management treatments. For prediction purposes, the selected stand density variable should have a strong correlation with the growth and yield variable of interest in the presence of the site index and stand age (Burkhart 2013). As a measure of density, the strength of SDI is that it takes into account the number per unit area and mean diameter (Eugene and Burkhart 1983, Ducey and Larson 1999, Zeide 2004). Zeide (2005) suggested SDI was one of the simplest and best measures for describing the stand density of pure stands. The SDI has been applied in many forest growth models because of its simplicity (e.g., Dean and Baldwin 1993, Pretzsch 2002, Monserud et al. 2005, Burkhart 2013, Rahman et al. 2016). SDI has been applied in biomass models (Dahlhausen et al. 2017) and process-based models (Landsberg and Waring 1997) as well. Therefore, an accurate estimate of SDI is critical if forest managers are to make realistic projections of forest development under different management scenarios (Poage et al. 2007).

Chinese fir (Cunninghamia lanceolata [Lamb.] Hook.) is one of the most important native tree species to timber supply in southern China. It has been planted extensively for over a thousand years because of its high-quality timber (i.e., decay-resistant wood and straight shape). It occupies 28.54 percent of all forested land in China, covering 9.215 million hectares (Zhang et al. 2013). Zhang et al. (2016) applied segmented regression to analyze the self-thinning line for Chinese fir stands and found that the slope of the self-thinning line was much steeper than Reineke's slope of $-1.605$. Comeau et al. (2010) suggested that climate resulted in differences in maximum density for Douglas-fir (Pseudotsuga menziesii) and Sitka spruce (Picea sitchensis). Brunet-Navarro et al. (2016) reported that the slopes of self-thinning were species-specific and affected by climatic conditions. Also, Condé et al. (2017) detected that climate influenced the maximum stocking in Scots pine (Pinus sylvestris L.) and European beech (Fagus sylvatica L.) stands. Zhang et al. (2018) incorporated climate variables to the segmented regression model and found that slopes of self-thinning lines for Chinese fir were not invariant and sensitive to climate variables.

The objective of this study was (1) to modify SDI_R by applying the self-thinning slopes by Zhang et al. (2016, 2018) and (2) to evaluate the incorporation of the modified SDIs into models to predict growth and survival of Chinese fir plantations across the south of China. The findings will be useful for forest managers to predict future growth for Chinese fir plantations under climate change.

Materials and Methods

Study Sites and Data

Data for this study were from Chinese fir stands established using bare-root seedlings that were planted in 1982 in Fujian, Guangxi, and Sichuan, and in 1981 in Jiangxi Province. The study sites located in Fujian, Jiangxi, and Sichuan Province belong to middle-subtropical climate zones. However, in Guangxi, the study site has a southern-subtropical climate (Figure 1, Table 1). Chinese fir is distributed mainly in the subtropical zone in southern China. At each site, the plots were installed with a randomized block design with four planting densities as follows: 2 m × 1.5 m (3,333 trees/hectare), 2 m × 1 m (5,000 trees/hectare), 1 m × 1.5 m (6,667 trees/hectare), and 1 m × 1 m (10,000 trees/hectare). Each density level was repeated three times. The size of each plot was 0.06 hectares, and there was a buffer zone that consisted of two rows of similarly treated trees surrounding each plot. At breast height, the diameters (dbh) of all trees in the plots were measured after the tree heights reached 1.3 m during the first winter. Also, they were subsequently measured every 2 or 3 years. In addition, the total tree height of more than 50 trees in each plot was measured. For each plot, the stand dominant height ($H$) was computed by taking the average height of the tallest six trees. There was a snow storm in Jiangxi in the winter of 1998 that resulted in a fair amount of mortality. Therefore, the data after 1999 from Jiangxi were excluded. The summary statistics for stand variables by site are described in Table 2, and the density-size trajectories of number of trees per hectare ($N$) and $Dq$ are shown by site in Figure 2.

In addition, the climate variables were obtained using the ClimateAP program (Wang et al. 2012). They mainly were mean annual temperature (MAT), annual precipitation (AP), mean warmest monthly temperature, mean coldest monthly temperature, degree-days below 0°C (DD_0), summer mean maximum temperature (SMMT), winter mean minimum temperature (WMMT), and spring mean temperature. The annual heat–moisture index ($AHM = [MAT + 10] / [AP / 1,000]$), an inverse form of the De Martonne aridity index (De Martonne 1926), was used to indicate the annual heat and water supply. Detailed climate information can be found in Zhang et al. (2018).

SDI

Reineke (1933) showed there was a linear relation (on a log–log scale) between the number of trees per unit area ($N$) and $Dq$ in even-aged, fully stocked stands as follows:

$$\ln N = k_0 + c \ln Dq,$$

where ln(·) is the natural logarithm, and $c$ is the slope, which was a common fixed value of $-1.605$ proposed by Reineke (1933) for most tree species, independent of age, site or initial density.

Management and Policy Implications

Stand density index (SDI) has played an important role in controlling stand stocking and modeling stand development in forest stands. Reineke's SDI (SDI_R), based on a constant slope of $-1.605$ for the self-thinning line, is questionable. For Chinese fir plantations, however, it has been reported that the self-thinning slope varied with site and climate. Remeasured data from 48 plots distributed in Fujian, Jiangxi, Guangxi, and Sichuan provinces were used to develop models for prediction of stand survival and basal area, with SDI_R, site-specific SDI (SDI_S), and climate-sensitive SDI (SDI_C), respectively, incorporated as a predictor variable. Results indicated that models with SDI_C performed best, followed by SDI_S and SDI_R. Incorporating SDI_C to forest growth and survival models is crucial for predicting future growth in response to climate change. Therefore, more attention should be paid to these factors when we apply SDI in models to predict future forest growth after controlling the level of growing stock of Chinese fir through either initial spacing or subsequent thinnings.
The SDI expresses the density of stands using $Dq$ and $N$ by determining the number of stems per hectare in these stands at an index diameter of 10 in. (25.4 cm). The SDI, as defined in China based on an index diameter of 20 cm, is used here:

$$SDI = N \left( \frac{20}{Dq} \right)^c$$

(2)

In this study, the first type of stand density index was SDI_R, which was based on $c = -1.605$. However, Zhang et al. (2016) suggested that the slope $c$ was not constant and used the following non-linear mixed-effects segment model to describe the self-thinning trajectories of Chinese fir:

$$y_2 = y_1 + (b + \nu) \left\{ (x_2 - a_{11} - a_{12}y_0)^2 I_{22} - \left( x_2 - a_{11} - a_{12}y_0 - \frac{c}{2b} \right)^2 I_{22} \right\} I_{22} - (x_1 - a_{11} - a_{12}y_0)^2 I_{11} + \left( x_1 - a_{11} - a_{12}y_0 - \frac{c}{2b} \right)^2 I_{11} + \varepsilon$$

(3)

where

$$I_{11} = \begin{cases} 1, \text{ if } x_i > a_{11} + a_{12}y_0, \\ 0, \text{ otherwise} \end{cases} \quad \text{and} \quad I_{22} = \begin{cases} 1, \text{ if } x_i > a_{11} + a_{12}y_0 + \frac{c}{2b}, \\ 0, \text{ otherwise} \end{cases} \quad i = 1, 2$$
found that the slope values of the self-thinning lines for Chinese (2018) used the following equation to reparameterize slope Navarro et al. (2016), Kweon and Comeau (2017). Zhang et al. (2016), labeled by SDI_S. Guangxi, and –4.2169 for Sichuan. Therefore, the second type of that the slope on the segment regression (Equation 3), Zhang et al. (2016) found climate variables (Zhang et al. 2018).

thinning slopes (ranging from –4.1647 to –1.6355) estimated from the last type of stand density index, SDI_C, was based on local self-
specific to each individual plot and assumed to be normally distrib-
Site plantations by site.

Figure 2. Density-size trajectories (ln Y-ln Dq) observed from 48 permanent plots in four planting densities stands of Chinese fir plantations by site.

\[
\begin{align*}
\hat{\ln}(Y) &= y_1 + (b + v) \left\{ (x_2 - a_{11} - a_{12}y_0)^2 I_{12} \\
&- (x_1 - a_{12}y_0)^2 I_{21} \\
&- (x_1 - a_{11})^2 I_{11} \\
&+ (x_1 - a_{11} - a_{12}y_0 - \frac{c_2}{2})^2 I_{21} \right\} + \epsilon_f \\
&= c_0 + c_1 \times \text{MAT} + c_2 \times \text{AP} + c_3 \\
&\times \text{DD}_0 + c_4 \times \text{SMMT} + c_5 \times \text{WMMT} \\
\end{align*}
\]

where \(c_0 - c_5\) are the parameters to be estimated. So, in this study, the last type of stand density index, SDI_C, was based on local self-thinning slopes (ranging from –4.1647 to –1.6355) estimated from climate variables (Zhang et al. 2018).

**Table 2. Summary statistics of stand variables of Chinese fir plantations by site.**

<table>
<thead>
<tr>
<th>Site</th>
<th>n</th>
<th>A</th>
<th>Mean</th>
<th>SD</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
<th>B</th>
<th>Mean</th>
<th>SD</th>
<th>Dq</th>
<th>Mean</th>
<th>SD</th>
<th>H</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fujian</td>
<td>192</td>
<td>15</td>
<td>8.05</td>
<td>5.12</td>
<td>2,262</td>
<td>45.79</td>
<td>20.72</td>
<td>11.26</td>
<td>4.35</td>
<td>13.77</td>
<td>6.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jiangxi</td>
<td>124</td>
<td>12</td>
<td>4.60</td>
<td>5.87</td>
<td>2,180</td>
<td>38.07</td>
<td>13.55</td>
<td>9.35</td>
<td>2.34</td>
<td>10.83</td>
<td>2.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guangxi</td>
<td>132</td>
<td>19</td>
<td>6.34</td>
<td>3.62</td>
<td>1,815</td>
<td>34.11</td>
<td>6.79</td>
<td>11.83</td>
<td>2.90</td>
<td>15.25</td>
<td>3.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sichuan</td>
<td>192</td>
<td>15</td>
<td>7.20</td>
<td>4.64</td>
<td>2,212</td>
<td>30.25</td>
<td>8.57</td>
<td>9.70</td>
<td>2.42</td>
<td>11.54</td>
<td>3.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: A, stand age; B, stand basal area (m²/hectare); Dq, stand quadratic mean of diameter (cm); H, stand dominant height (m); n, total number of measurements across all plots; N, surviving number of trees per hectare; SD, standard deviation.*

**Stand Growth and Survival Models**

The stand basal area and survival models for predicting growth of Chinese fir were developed in this study. Dominant height (H), a component of both models, was modeled separately using Bailey and Clutter’s (1974) equation:

\[
H_{t+q} = \exp\{\phi_1 + [\ln(H_t – \phi_1)](A_{t+q}/A_t)^{\phi_2}\} + \varepsilon
\]

where \(H_t\) and \(H_{t+q}\) are the dominant heights at ages \(A_t\) and \(A_{t+q}\), respectively; \(q\) is the growth period length in years; \(\phi_1, \phi_2\) are parameters; and \(\varepsilon\) is the random error.

Annual growth projection is a growth-modeling method that can account for different growth intervals and maintain step invariance of model outputs (Ochi and Cao 2003). The recursive manner of this method was suitable for the data in this study that had variable growth intervals.

Year (\(t+1\)):

\[
\hat{N}_{t+1} = N_t / [1 + \exp(\alpha_1 + \alpha_2/A_t + \alpha_3/\ln(SDI_t))]
\]

\[
\hat{Dq}_{t+1} = Dq_t + \exp[\beta_1 + \beta_2/\ln(H_t) + \beta_3/\ln(SDI_t)]
\]

\[
\hat{B}_{t+1} = (\pi/40,000)\hat{N}_{t+1}\hat{Dq}^2_{t+1}
\]

Year (\(t+q\)):

\[
\hat{N}_{t+q} = \hat{N}_{t+q-1} / [1 + \exp(\alpha_1 + \alpha_2/A_t + \alpha_3/\ln(SDI_{t+q-1}))]
\]

\[
\hat{Dq}_{t+q} = \hat{Dq}_{t+q-1} + \exp[\beta_1 + \beta_2/\ln(H_{t+q-1}) + \beta_3/\ln(SDI_{t+q-1})]
\]

\[
\hat{B}_{t+q} = (\pi/40,000)\hat{N}_{t+q}\hat{Dq}^2_{t+q}
\]

where \(N, Dq,\) and \(B\) are the observed stand survival per hectares, quadratic mean diameter in centimetres, and stand basal area in square metres/hectare, respectively (subscripts denoting measurement time); the \(^\wedge\) symbol denotes the predicted value; SDI is SDI_R, SDI_S, or SDI_C; and the \(\alpha\) and \(\beta\) values are the parameters to be estimated.

We also developed growth models without SDI (labeled SDI_no), to be used as control models for comparison against the models with SDIs. Because there existed cross-equation correlations among the error components of these models, the seemingly unrelated regression method was employed to estimate coefficients of the above system using the procedure NLMIXED in SAS (SAS Institute 2011).
Model Evaluation

The following evaluation statistics were used to validate the stand survival and stand basal area models:

\[
\text{Mean difference (MD)}: \quad \text{MD} = \frac{\sum_i (y_i - \hat{y}_i)}{n}
\]

(12)

\[
\text{Mean absolute difference (MAD)}: \quad \text{MAD} = \frac{\sum_i |y_i - \hat{y}_i|}{n}
\]

(13)

\[
\text{Fit index: } R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}
\]

(14)

where \(y_i\) and \(\hat{y}_i\) are the observed and predicted stand survival or stand basal area for the \(i\)th observation, respectively; \(n\) is the total number of samples; and the summation sign includes values of \(i\) from 1 to \(n\). The model with lower absolute values of MD, lower MAD values, and larger \(R^2\) values indicates that the model performed better on fitting the data.

In order to describe the relative position of each competition index, we employed Poudel and Cao's (2013) relative rank system. The best and worst methods receive relative ranks of 1 and \(m\), respectively, in this ranking system of \(m\) methods. The remaining methods are ranked as real numbers between 1 and \(m\). This ranking system should provide more information than the traditional ordinal ranks because the magnitudes as well as the order of evaluation statistic are taken into consideration.

A two-step approach was carried out. First, a relative rank was calculated for each statistic using each method. Then, a final rank was determined based on the sum of all three ranks for each method.

Results and Discussions

The parameter estimates (and standard errors) of the dominant height growth model were \(\hat{\alpha}_1 = 4.1277 (0.2982)\) and \(\hat{\phi}_1 = -0.4270 (0.0814)\). The resulting \(R^2\) value of 0.9567 indicates that Equation 5 fitted the dominant height growth data well. All of the parameters were significant at the 0.05 level (Table 3).

The evaluation statistics output from the four methods is listed in Table 4. These results showed that the performance of growth models that contained an SDI depended on how well the self-thinning slope, used by that SDI, fitted the data. The overall ranks of the models were in the following order: SDI_C (1.00), SDI_S (1.54), SDI_R (2.74), and SDI_No (4.00).

The models without SDI were last in terms of \(R^2\) for stand survival and basal area prediction, and also last in terms of MAD for basal area. Overall, they ranked last in predicting stand survival and basal area, and were clearly inferior to the models that incorporated SDIs.

Because it was based on a generic self-thinning slope of –1.605, SDI_R did not perform as well as the other two SDIs. Except for ranking first in terms of MD for stand survival, SDI_R consistently produced the worst evaluation statistics among the three SDIs considered in this study. Although Reineke (1933) reported that the slope of the self-thinning line was –1.605 for 12 of 14 species, other reports indicated that the slope varied by tree species and sites (Jack and Long 1996, Bi and Turvey 1997, Weiskittel et al. 2009). Poage et al. (2007) reported that the slope of individual stands from 40 hemlock (Tsuga heterophylla [Raf.] Sarg.)–Sitka spruce stands ranged from –2.192 to –1.310. Comeau et al. (2010) found that in the UK, the slopes of Douglas-fir and Sitka spruce were –2.063 and –1.864 respectively. However, slopes were lower in Western Canada (–1.437 for Sitka spruce and –1.241 for Douglas-fir) than in the UK. Charru et al. (2012) found that slopes were species-specific, ranging from –1.941 to –1.615 for 11 temperate species in France.

When the self-thinning slope was site-specific to Chinese fir plantations, the corresponding stand density index (SDI_S) brought forth improved performance compared to SDI_R. Instead of –1.605, SDI_S was based on a slope of –2.4453 for Fujian, Jiangxi, and Guangxi, and –4.2169 for Sichuan. For a particular stand, the SDI value computed from –1.605 has to be greater than the SDI values based on steeper slopes, resulting in an artificially higher
competition. As a result, the growth models with SDI_S scored better than those with SDI_R for most of the evaluation statistics.

Whereas SDI_S was computed from a separate self-thinning slope for each of the two groups of sites (one group is Fujian, Jiangxi, and Guangxi, and the second is Sichuan), SDI_C was based on slopes that varied depending on climate variables for each locality. The self-thinning slope, $c$, ranged from –4.1647 to –1.6355. The SDI founded on climate-sensitive self-thinning slopes was site-specific and therefore ranked first overall, and first in all but one of the evaluation statistics.

Although Zhang et al. (2018) found that MAT and DD_0 affected the slopes of self-thinning line significantly, the SDI_C showed no significant correlation with MAT and DD_0 ($P > .05$, Figure 3). However, SDI_C significantly increased with increasing AP and SMMT. In most studies, tree growth was positively correlated with AP (e.g., Chhin et al. 2008, Aubry-Kientz and Moran 2017, Żywiec et al. 2017), resulting in an enhanced ability to adapt to climate change. Kharuk et al. (2013) reported that climate-induced forest density increased with increasing AP. In contrast, SDI_C decreased with increasing WMMT and AHM ($P < .01$, Figure 3), which indicated that a higher mortality was related to a higher AHM. This result confirmed the findings of van Mantgem et al. 2009, Peng et al. (2011), and Zhang et al. (2014, 2017), who showed that water stress, which was induced by climate change, increased tree mortality.

The SDI is a crucial dynamic attribute that can be used to assess full-site occupancy in a stand. It has been shown to be valuable for advancing the understanding of stand density effects on forest-stand development (Burkhart 2013). As elaborated by Zeide (2005), the SDI might be the most significant contribution from Americans to forest science. However, SDI_R has been questioned because it is based on a self-thinning line with a constant slope of –1.605 (Pretzsch and Biber 2005). Pretzsch and Biber (2005) quoted SDI values in the yield tables for Norway spruce ($Picea abies$ [L.] Karst.) by Assmann and Franz (1965), and found that assuming a constant slope of –1.605 would result in biased SDI values, whereas using the actual slope of –1.805 would reduce the bias. Luis and Fonseca (2004) have suggested the use of an SDI with a slope of –1.897 instead of –1.605 for stand stocking regulation. Pretzsch (2005) calculated the standardized stand density index (SSDI) with experiment-specific slope values of Norway spruce and European beech ($Fagus sylvatica$ L.) instead of –1.605, and incorporated the SSDI into growth models. Burkhart (2013) modified SDI by replacing quadratic mean diameter with dominant height or mean tree volume and reported that the model with the Reineke-defined SDI calculated from a slope of –1.4309 performed best.

Figure 3. Correlation plots ($R$: correlation coefficient) of SDI_C values versus climate variables, including annual precipitation (AP, $R = .7225$), annual heat–moisture index (AHM, $R = .6584$), winter mean minimum temperature (WMMT, $R = –.2145$), summer mean maximum temperature (SMMT, $R = .3833$), mean annual temperature (MAT, $R = –.0360$), and degree-days below 0° C (DD_0, $R = –.0378$).
It therefore makes sense that SDI should depend on how well the slope fits the data at the self-thinning stage. Results from this study showed that using SDI computed from a climate-sensitive self-thinning slope, which was specific to that locality, would provide better predictions for stand survival and basal area.

Incorporating SDI_C into forest growth and survival models is crucial for predicting future growth in response to climate change. The results suggest that the SDI values of Chinese fir were sensitive to climate variables and incorporating the climate-sensitive SDI into growth models improved the model performance. Therefore, more attention should be focused on these factors when we apply SDI in models to predict future forest growth after controlling the level of growing stock of Chinese fir through either initial spacing or subsequent thinnings.

Conclusions

In this study, we modified SDI_R using the self-thinning slope for Chinese fir plantations reported by Zhang et al. (2016) and climate-sensitive slope from Zhang et al. (2018). The SDIs were incorporated into regression models to predict stand survival and basal area. Results indicated that models with climate-sensitive SDI (SDI_C) performed best, followed by SDI_S and SDI_R. The models without SDI (SDI_no) received the worst overall rank as expected. Overall, inclusion of climate-sensitive SDI in growth and survival models can facilitate projection of the growth/survival-stand density relation under future climate-change conditions.

Literature Cited


